The NQL-Package

A Nilpotent Quotient Algorithm for L-presented Groups

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GAP Package Authors Workshop 2007

L-presentations and L-presented groups

Definition (Bartholdi, 2003)

A (finite) *L-presentation* (or *endomorphic presentation*) is an expression of the form

$$\langle \mathcal{S} \mid \mathcal{Q} \mid \Phi \mid \mathcal{R} \rangle,$$

where $\mathcal{S}$ is a (finite) alphabet, $\mathcal{Q}$ and $\mathcal{R}$ are (finite) subsets of the free group F on $\mathcal{S}$, and Φ is a (finite) set of endomorphisms of F.

L-presentations and L-presented groups

Definition (Bartholdi, 2003)

A (finite) L-presentation $\langle \mathcal{S} \mid \mathcal{Q} \mid \Phi \mid \mathcal{R} \rangle$ defines the *(finitely) L-presented group* $G = F/K$, where

$$K = \left\langle \mathcal{Q} \cup \bigcup_{\sigma \in \Phi^*} \sigma(\mathcal{R}) \right\rangle^F$$

and Φ^* is the monoid generated by Φ.

L-presentations and L-presented groups

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An L-presentation $\langle \mathcal{S} \mid \mathcal{Q} \mid \Phi \mid \mathcal{R} \rangle$ is called *invariant*, if $K = \langle \mathcal{Q} \cup \bigcup_{\sigma \in \Phi^*} \sigma(\mathcal{R}) \rangle^F$ satisfies $\sigma(K) \subseteq K$ for each $\sigma \in \Phi$.

Each L-presentation of the form $\langle \mathcal{S} \mid \emptyset \mid \Phi \mid \mathcal{R} \rangle$ is invariant.

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Remark

Each finite presentation $\langle \mathcal{X} \mid \mathcal{R} \rangle$ translates to an invariant L-presentation of the form $\langle \mathcal{X} \mid \emptyset \mid \{\mathsf{id}\} \mid \mathcal{R} \rangle$.

$\Rightarrow$(invariant) L-presentations generalize finite presentations

Examples of L-presented groups

Lysënok: The Grigorchuk Group has an L-presentation

$$\langle a, b, c, d \mid a^2, b^2, c^2, d^2, bcd \mid \sigma \mid [d, d^a], [d, d^{acaca}] \rangle$$

where σ is a free group homomorphism induced by

$$\sigma \colon F \to F : \begin{cases} a & \mapsto & c^a \\ b & \mapsto & d \\ c & \mapsto & b \\ d & \mapsto & c \end{cases} .$$

Examples of L-presented groups

Further finitely L-presented groups (not finitely presented)

- Gupta-Sidki Group and some generalizations
- Brunner-Sidki-Vieira Group
- Basilica Group
- Fabrykowski-Gupta Group and some generalizations

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Theorem (Bartholdi, 2007)

Each finitely generated normal subgroup of a finitely presented group is finitely L-presented.

Polycyclic Presentations

Definition (PcpGroups)

A *polycyclic presentation* is a finite presentation on $a_1, \ldots, a_n$, say, with relations of the form

$$\begin{array}{rcll} a_j^{a_i} & = & u_{ij}(a_{i+1}, \ldots, a_n) & \text{for } i < j \\ a_j^{a_i^{-1}} & = & v_{ij}(a_{i+1}, \ldots, a_n) & \text{for } i < j, r_i = \infty \\ a_i^{r_i} & = & w_{ii}(a_{i+1}, \ldots, a_n) & \text{if } r_i < \infty \end{array}$$

for certain $r_1, \ldots, r_n \in \mathbb{N} \cup \{\infty\}$.

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for certain $r_1, \ldots, r_n \in \mathbb{N} \cup \{\infty\}$.

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Polycyclic presentations allow effective computations.

Nilpotent Quotient Algorithm

Aim: Compute polycyclic presentations for the lower central series quotients $G/\gamma_{c+1}(G)$ for a given c.

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- $\rightsquigarrow$ read off the abelian invariants of $\gamma_i(G)/\gamma_{i+1}(G)$ for $i \leq c$
- $\rightsquigarrow$ verify whether G has a maximal nilpotent quotient
- $\rightsquigarrow$ read off other properties of $G/\gamma_{c+1}(G)$

The Abelian Quotient (case $c = 2$)

Let $G = F/K$ with $K = \langle \mathcal{Q} \cup \bigcup_{\sigma\in\Phi^*} \sigma(\mathcal{R})\rangle^F$.

1. Start with $F/F' \cong \mathbb{Z}^m$ for $m = \mathrm{rk}(F)$
2. Translate $\sigma \in \Phi$ to $M_\sigma \in \mathbb{Z}^{m\times m}$
3. Translate $g \in \mathcal{Q} \cup \mathcal{R}$ to $\bar{g} \in \mathbb{Z}^m$
4. Let $U = \langle \bar{q}, \bar{r}M_\sigma \mid q \in \mathcal{Q}, r \in \mathcal{R}, \sigma \in \Phi^* \rangle$
5. Determine a finite subgroup basis of U

$\rightsquigarrow$ read off a polycyclic presentation for $G/G' \cong \mathbb{Z}^m/U$.

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1. Reduce to invariant L-presentations
2. For invariant L-presentations use induction on c

   - ⇝ generalize the nilpotent quotient algorithm for finitely presented groups as implemented in the NQ-Package (W. Nickel, 1995)
   - ⇝ explicit algorithm is rather technical; it uses ideas similar to those for the abelian quotient

Brunner-Sidki-Vieira Group

Brunner, Sidki, and Vieira, *A just-non-solvable torsionfree group defined on the binary tree.* 1999.

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A group with invariant L-presentation

$$G = \langle \lambda, \tau \mid \emptyset \mid \sigma \mid [\lambda, \lambda^{\tau}], [\lambda, \lambda^{\tau^3}] \rangle$$

where σ is induced by $\tau \mapsto \tau^2$ and $\lambda \mapsto \tau^2 \lambda^{-1} \tau^2$.

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where σ is induced by $\tau \mapsto \tau^2$ and $\lambda \mapsto \tau^2 \lambda^{-1} \tau^2$.

So far G/G' and $G'/\gamma_3(G)$ are known.

Our algorithm: $\gamma_i(G)/\gamma_{i+1}(G)$ for $i \leq 50$.

Brunner-Sidki-Vieira Group

i	Abelian invariants of $\gamma_i(G)/\gamma_{i+1}(G)$
1, . . . ,3	(0,0),(0),(8)
4, . . . ,6	(8),(4,8),(2,8)
7, . . . ,12	(2,2,8),(2,2,8),(2,2,4,8),(2,2,4,8),(2,2,2,8),(2,2,2,8)

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$7,\dots,12$	$(2,2,8)^{[2]}, (2,2,4,8)^{[2]}, (2,2,2,8)^{[2]}$

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$7,\ldots,12$	$(2^{[2]},8)^{[2]},(2^{[2]},4,8)^{[2]},(2^{[2+1]},8)^{[2]},$

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$7,\ldots,12$	$(2^{[2]},8)^{[2]}, (2^{[2]},4{,}8)^{[2]}, (2^{[2+1]},8)^{[2]},$
$14,\ldots,24$	$(2^{[4]},8)^{[4]}, (2^{[4]},4{,}8)^{[4]}, (2^{[4+1]},8)^{[4]},$
$25,\ldots,48$	$(2^{[6]},8)^{[8]}, (2^{[6]},4{,}8)^{[8]}, (2^{[6+1]},8)^{[8]}$
$49,\ldots,50$	$(2^{[8]},8)^{[2]}$

Brunner-Sidki-Vieira Group

Conjecture

The abelian invariants of $\gamma_i(G)/\gamma_{i+1}(G)$, $i \geq 4$ *are*

$$\begin{array}{rl} (2^{[2k]}, 8) & \text{if } i \in \{3 \cdot 2^k + 1, \ldots, 4 \cdot 2^k\} \\ (2^{[2k]}, 4, 8) & \text{if } i \in \{4 \cdot 2^k + 1, \ldots, 5 \cdot 2^k\} \\ (2^{[2k+1]}, 8) & \text{if } i \in \{5 \cdot 2^k + 1, \ldots, 6 \cdot 2^k\} \end{array}$$

for $k \in \mathbb{N}_0$.

Basilica Group

Grigorchuk & Żuk. *Spectral properties of a torsion-free weakly branch group defined by a three state automaton.* 2002.

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Bartholdi & Virág, 2005: An invariant L-presentation

$$\Delta = \langle a, b \mid \emptyset \mid \sigma \mid [b, b^a] \rangle$$

where σ is induced by $a \mapsto b$ and $b \mapsto a^2$.

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So far only the abelian quotient Δ/Δ' is known.

Our algorithm: $\gamma_i(\Delta)/\gamma_{i+1}(\Delta)$ for $i \leq 90$.

Basilica Group

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$1,\dots,6$	$(0,0),(0),(4)^{[2]},(4,4),(2,4)$
$7,\dots,12$	$(2^{[2]},4)^{[2]},(2^{[3]},4)^{[1]},(2^{[4]},4)^{[2]},(2^{[3]},4)^{[1]}$
$13,\dots,25$	$(2^{[4]},4)^{[4]},(2^{[5]},4)^{[2]},(2^{[6]},4)^{[4]},(2^{[5]},4)^{[2]}$
$26,\dots,48$	$(2^{[6]},4)^{[8]},(2^{[7]},4)^{[4]},(2^{[8]},4)^{[8]},(2^{[7]},4)^{[4]}$
$49,\dots,90$	$(2^{[8]},4)^{[16]},(2^{[9]},4)^{[8]},(2^{[10]},4)^{[16]},(2^{[9]},4)^{[2]}$

Basilica Group

Conjecture

The abelian invariants of $\gamma_i(\Delta)/\gamma_{i+1}(\Delta)$, $i \geq 7$ *are*

$$\begin{array}{ll} (2^{[2k+2]}, 4) & \text{if } i \in \{6 \cdot 2^k + 1, \dots, 8 \cdot 2^k\} \\ (2^{[2k+3]}, 4) & \text{if } i \in \{8 \cdot 2^k + 1, \dots, 9 \cdot 2^k\} \\ (2^{[2k+4]}, 4) & \text{if } i \in \{9 \cdot 2^k + 1, \dots, 11 \cdot 2^k\} \\ (2^{[2k+3]}, 4) & \text{if } i \in \{11 \cdot 2^k + 1, \dots, 12 \cdot 2^k\} \end{array}$$

for $k \in \mathbb{N}_0$.

Fabrykowski-Gupta Groups

Fabrykowski & Gupta. *On groups with sub-exponential growth functions.* 1985.

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Generalization (Bartholdi, 2007): For $n \geq 3$ let

$$\Gamma_n = \langle \alpha, \rho \mid \emptyset \mid \varphi \mid \mathcal{R} \rangle,$$

with $\sigma_i = \rho^{\alpha^i}$ for $1 \leq i \leq n$ and

$$\mathcal{R} = \left\{ \alpha^n, \left[\sigma_i^{\sigma_{i-1}^l}, \sigma_j^{\sigma_{j-1}^m} \right], \sigma_i^{-\sigma_{i-1}^{l+1}} \sigma_i^{\sigma_{i-1}^l \sigma_{i-1}^{\sigma_{i-2}^m}} \;\middle|\; \begin{array}{c} 1 \leq i,j \leq n, \\ 2 \leq |i-j| \leq n-2, \\ 0 \leq l,m \leq n-1 \end{array} \right\}$$

and φ is induced by $\alpha \mapsto \rho^{\alpha^{-1}}$ and $\rho \mapsto \rho$.

Fabrykowski-Gupta Groups (n prime)

If n is prime then $\gamma_i(\Gamma_n)/\gamma_{i+1}(\Gamma_n)$ are n-elementary abelian groups with n-ranks

- Γ_3: $2, 1, 2, 1, 2^{[3]}, 1^{[3]}, 2^{[9]}, 1^{[9]}, 2^{[27]}, 1^{[27]}$
- Γ_5: $2, 1^{[3]}, 2, 1^{[13]}, 2^{[5]}, 1^{[65]}, 2^{[8]}$
- Γ_7: $2, 1^{[5]}, 2, 1^{[33]}, 2^{[7]}, 1^{[27]}$
- Γ_{11}: $2, 1^{[9]}, 2, 1^{[54]}$

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- Γ_5: $2, 1^{[3]}, 2, 1^{[13]}, 2^{[5]}, 1^{[65]}, 2^{[8]}$
- Γ_7: $2, 1^{[5]}, 2, 1^{[33]}, 2^{[7]}, 1^{[27]}$
- Γ_{11}: $2, 1^{[9]}, 2, 1^{[54]}$

Conjecture

If n is an odd prime, then Γ_n is a group of width 2.

Fabrykowski-Gupta Groups (n prime-power)

If $n = p^k$ then $\gamma_i(\Gamma_n)/\gamma_{i+1}(\Gamma_n)$ are p-elementary abelian, except for some initial entries:

$\Gamma_4 : (4,4), (4), 2^{[4]}, 3^{[3]}, 2^{[13]}, 3^{[12]}, 2^{[52]}, 3^{[38]}$

$\Gamma_8 : (8,8), (8), (4)^{[4]}, 2, 1, 2^{[2]}, 3, 2, 3^{[2]}, 4, 3^{[8]}, 2^{[23]}, 3^{[5]}, 2^{[1]}$

$\Gamma_9 : (9,9), (9)^{[2]}, 1^{[5]}, 2^{[6]}, 3, 2^{[17]}, 1^{[38]}, 2^{[36]}$

Fabrykowski-Gupta Groups (n composite)

If $n \in \{6, 10, 12, 14, 15, 18, 20, 21\}$ then the groups Γ_n have a maximal nilpotent quotient.

Conjecture

If n is a composite, then Γ_n has a maximal nilpotent quotient.

Further experiments with

- Gupta-Sidki Group and some generalizations
- Grigorchuk Super Group from

  Bartholdi & Grigorchuk. *On parabolic subgroups of some fractal groups*. 2002.
- Baumslag. *A finitely generated, infinitely related group with trivial multiplicator*. 1971.

The algorithm is implemented in the GAP4 package NQL and described explicitly in

Eick, Hartung, Bartholdi. *A nilpotent quotient algorithm for L-presented groups*. 2007.