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# Investigating p-groups by coclass with Gap

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September 2007 GAP Workshop, Braunschweig



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## COCLASS GRAPH

## DEFINITION (LEEDHAM-GREEN, NEWMAN 1980)

A finite *p*-group *G* with  $|G| = p^n$  and cl(G) = c has coclass

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 $\operatorname{cc}(G) = n - c.$ 

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Notation:

• If  $\overline{GH}$  edge, then G is a descendant of H.

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- If  $\overline{GH}$  edge, then G is a descendant of H.
- If *G* has descendants, then *G* is called capable.

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Use the Coclass-Graph  $\mathcal{G}(p, r)$ : • vertices of  $\mathcal{G}(p, r) \leftrightarrow p$ -groups of coclass r, • edge  $\overline{GH} \leftrightarrow G/\gamma_c(G) \cong H$  with c = cl(G).

Notation:

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- If *G* has descendants, then *G* is called capable, otherwise terminal.

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#### INVESTIGATING EXAMPLE: THE GRAPH $\mathcal{G}(2, 1)$ **D**-GROUPS BY COCLASS WITH GAP $V_4$ $C_4$ ΗΕΙΚΟ 2<sup>2</sup> DIETRICH\*. BETTINA EICK, Dörte FEICHTEN-2<sup>3</sup> SCHLAGER\* $D_8$ $Q_8$ INTRODUCTION

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# Investigating $\rho$ -groups by EXAMPLE: THE GRAPH $\mathcal{G}(2, 1)$

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### MAIN CONJECTURE

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#### **CONJECTURE**

Let  $r \in \mathbb{N}$ . The *p*-groups of coclass *r* can be split into finitely many coclass families.

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### MAIN CONJECTURE

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#### CONJECTURE

Let  $r \in \mathbb{N}$ . The p-groups of coclass r can be split into finitely many coclass families such that

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• the groups in a family can be defined by a single parametrized presentation.

### MAIN CONJECTURE

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#### CONJECTURE

Let  $r \in \mathbb{N}$ . The p-groups of coclass r can be split into finitely many coclass families such that

- the groups in a family can be defined by a single parametrized presentation;
- many structural invariants of the groups in a family can be exhibited in a uniform way.

## MAIN CONJECTURE

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- the groups in a family can be defined by a single parametrized presentation;
- many structural invariants of the groups in a family can be exhibited in a uniform way, for example:
  - Schur multiplicators

can be described in a parametrized presentation.

## MAIN CONJECTURE

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- the groups in a family can be defined by a single parametrized presentation;
- many structural invariants of the groups in a family can be exhibited in a uniform way, for example:
  - Schur multiplicators and
  - automorphism groups

can be described in a parametrized presentation.

## MAIN CONJECTURE

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- the groups in a family can be defined by a single parametrized presentation;
- many structural invariants of the groups in a family can be exhibited in a uniform way, for example:
  - Schur multiplicators,
  - automorphism groups and
  - cohomology rings  $H^*(-, R)$ , for R ring,

can be described in a parametrized presentation.

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## THE COCLASS TREE

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## Let $G \in \mathcal{G}(p, r)$ . • $\mathcal{T}_G \subseteq \mathcal{G}(p, r)$ is the descendant tree of G.

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### THE COCLASS TREE

# Let $G \in \mathcal{G}(p, r)$ .

- $T_G \subseteq \mathcal{G}(p, r)$  is the descendant tree of G.
- $T_G$  is a coclass tree  $\Leftrightarrow$  it has exactly one infinite path  $(G = G_0, G_1, G_2...)$ , its mainline, and is maximal with this property.

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## THE COCLASS TREE

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- The *i*-th branch B<sub>i</sub> of T<sub>G</sub> is generated by all descendants of G<sub>i</sub> which are not descendants of G<sub>i+1</sub>.

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• Depth of  $\mathcal{B}_i$  = length of a longest path in  $\mathcal{B}_i$ .

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- The *i*-th branch B<sub>i</sub> of T<sub>G</sub> is generated by all descendants of G<sub>i</sub> which are not descendants of G<sub>i+1</sub>.
- Depth of  $\mathcal{B}_i$  = length of a longest path in  $\mathcal{B}_i$ .
- Width of  $\mathcal{B}_i$  = maximum number of groups of same order in  $\mathcal{B}_i$ .

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## CONSTRUCTION RULES AND COCLASS FAMILIES

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#### **CONJECTURE**

 $\mathcal{T}$  coclass tree  $\mathcal{G}(p, r)$ .  $\Rightarrow$  Exist  $d, f \in \mathbb{N}$  such that

•  $\mathcal{B}_{i+d}$  can be constructed from  $\mathcal{B}_i$  for  $i \ge f$ .
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## CONSTRUCTION RULES AND COCLASS FAMILIES

### **CONJECTURE**

 $\mathcal T$  coclass tree  $\mathcal G(p,r). \Rightarrow$  Exist d,  $f \in \mathbb N$  such that

- $\mathcal{B}_{i+d}$  can be constructed from  $\mathcal{B}_i$  for  $i \ge f$ ;
- we get a surjective map  $\varphi_i : \mathcal{B}_{i+d} \to \mathcal{B}_i$  for  $i \ge f$ .

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### CONJECTURE

 $\mathcal T$  coclass tree  $\mathcal G(p,r). \Rightarrow$  Exist d,  $f \in \mathbb N$  such that

- $\mathcal{B}_{i+d}$  can be constructed from  $\mathcal{B}_i$  for  $i \ge f$ ;
- we get a surjective map  $\varphi_i : \mathcal{B}_{i+d} \to \mathcal{B}_i$  for  $i \ge f$ .

Choose suitable  $m \ge f$  and  $G \in B_i$  with  $m \le i < m + d$ .

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## CONSTRUCTION RULES AND COCLASS FAMILIES

### CONJECTURE

 $\mathcal{T}$  coclass tree  $\mathcal{G}(p, r)$ .  $\Rightarrow$  Exist  $d, f \in \mathbb{N}$  such that

- $\mathcal{B}_{i+d}$  can be constructed from  $\mathcal{B}_i$  for  $i \ge f$ ;
- we get a surjective map  $\varphi_i : \mathcal{B}_{i+d} \to \mathcal{B}_i$  for  $i \ge f$ .

Choose suitable  $m \ge f$  and  $G \in \mathcal{B}_i$  with  $m \le i < m + d$ . Then *G* defines an infinite coclass family  $\mathcal{F}_G$  consisting of *G* and iterated preimages of *G* under  $\varphi_{i+dj}$ , for  $j \in \mathbb{N}$ .

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## CONSTRUCTION RULES AND COCLASS FAMILIES

### CONJECTURE

 $\mathcal T$  coclass tree  $\mathcal G(p,r).\Rightarrow$  Exist  $d,f\in\mathbb N$  such that

- $\mathcal{B}_{i+d}$  can be constructed from  $\mathcal{B}_i$  for  $i \ge f$ ;
- we get a surjective map  $\varphi_i : \mathcal{B}_{i+d} \to \mathcal{B}_i$  for  $i \ge f$ .

Choose suitable  $m \ge f$  and  $G \in \mathcal{B}_i$  with  $m \le i < m + d$ . Then *G* defines an infinite coclass family  $\mathcal{F}_G$  consisting of *G* and iterated preimages of *G* under  $\varphi_{i+di}$ , for  $j \in \mathbb{N}$ .

### THEOREM (EICK, LEEDHAM-GREEN)

Let  $\mathcal{T}$  be a bounded coclass tree. Then there exist  $d, f \in \mathbb{N}$  and isomorphisms  $\mathcal{B}_{i+d} \to \mathcal{B}_i, i \ge f$ .

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# EXAMPLE: $\mathcal{G}(2,1)$ and $\mathcal{G}(2,2)$

 $\mathcal{G}(2,1)$  contains 6 coclass families.

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# EXAMPLE: $\mathcal{G}(2,1)$ and $\mathcal{G}(2,2)$

 $\mathcal{G}(2,1)$  contains 6 coclass families:

• 3 finite families (containing  $C_4$ ,  $V_4$  and  $Q_8$ , resp.).

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 $\mathcal{G}(2,1)$  contains 6 coclass families:

- 3 finite families (containing  $C_4$ ,  $V_4$  and  $Q_8$ , resp.) and
- 3 infinite families (dihedral, quaternion, semi-dihedral).

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 $\mathcal{G}(2,2)$  contains

• 5 coclass trees  $\mathcal{T}_1(2,2),\ldots,\mathcal{T}_5(2,2)$  .

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 $\mathcal{G}(2,2)$  contains

- 5 coclass trees  $\mathcal{T}_1(2,2),\ldots,\mathcal{T}_5(2,2)$  ,
- 19 finite families (each contains one group) .

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 $\mathcal{G}(2,2)$  contains

- 5 coclass trees  $\mathcal{T}_1(2,2),\ldots,\mathcal{T}_5(2,2)$  ,
- 19 finite families (each contains one group),
- 51 infinite families.

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# EXAMPLE: $\mathcal{G}(2,1)$ and $\mathcal{G}(2,2)$

- $\mathcal{G}(2,1)$  contains 6 coclass families:
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  - 3 infinite families (dihedral, quaternion, semi-dihedral).
- $\mathcal{G}(2,2)$  contains
  - 5 coclass trees  $\mathcal{T}_1(2,2),\ldots,\mathcal{T}_5(2,2)$  ,
  - 19 finite families (each contains one group),
  - 51 infinite families.

	$T_1(2,2)$	$T_2(2,2)$	$T_3(2,2)$	$T_4(2,2)$	$T_5(2,2)$
dim	2	2	1	1	1
$G_0$	(64,34)	(64,32)	(16,4)	(32,9)	(8,5)
# fam.	19	16	4	6	6
$G_{f}$	(64,34)	(64,32)	(16,4)	(32,9)	(16,11)

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## PRO-*p*-GROUPS OF COCLASS *r*

### THEOREM

There is a 1–1 correspondence between the pro-p-groups of coclass r and the mainlines of coclass trees in  $\mathcal{G}(p, r)$ .

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## PRO-*p*-GROUPS OF COCLASS *r*

### THEOREM

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### DEFINITION

A uniserial *p*-adic space group of dimension *d* is an extension.

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## PRO-*p*-GROUPS OF COCLASS *r*

### THEOREM

There is a 1–1 correspondence between the pro-p-groups of coclass r and the mainlines of coclass trees in  $\mathcal{G}(p, r)$ .

### DEFINITION

A uniserial *p*-adic space group of dimension *d* is an extension of

• a translation subgroup  $T = \mathbb{Z}_p^d$  ( $\mathbb{Z}_p$  *p*-adic integers).

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### THEOREM

There is a 1–1 correspondence between the pro-p-groups of coclass r and the mainlines of coclass trees in  $\mathcal{G}(p, r)$ .

### DEFINITION

A uniserial p-adic space group of dimension d is an extension of

PRO-*p*-GROUPS OF COCLASS *r* 

- a translation subgroup  $T = \mathbb{Z}_{\rho}^{d}$  ( $\mathbb{Z}_{\rho}$  *p*-adic integers) by
- a point group *P* (finite *p*-group) acting uniserially on *T*.

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### THEOREM

There is a 1–1 correspondence between the pro-p-groups of coclass r and the mainlines of coclass trees in  $\mathcal{G}(p, r)$ .

### DEFINITION

A uniserial p-adic space group of dimension d is an extension of

- a translation subgroup  $T = \mathbb{Z}_p^d$  ( $\mathbb{Z}_p$  *p*-adic integers) by
- a point group P (finite p-group) acting uniserially on T;
  i.e. T<sub>0</sub> = T and T<sub>i+1</sub> = [T<sub>i</sub>, P] satisfies [T<sub>i</sub> : T<sub>i+1</sub>] = p for all *i*.

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### PRO-*p*-GROUPS OF COCLASS *r*

## NUMBER OF PRO-*p*-GROUPS

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### Number of uniserial *p*-adic space groups:

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p = 2: Newman, O'Brien

*p* odd: construction algorithm by Eick

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### Number of uniserial *p*-adic space groups:

p = 2: Newman, O'Brien p odd: construction algorithm by Eick

	<i>r</i> = 2	<i>r</i> = 3	<i>r</i> = 4
<i>p</i> = 2	2	22	
<i>p</i> = 3	10	1271	137299952383
p = 5	95	1110136753555665	
p = 7	4575		

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## NUMBER OF PRO-*p*-GROUPS

## Number of uniserial *p*-adic space groups:

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	r = 2	<i>r</i> = 3	<i>r</i> = 4
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## Number of pro-*p*-groups:

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# NUMBER OF PRO-*p*-groups

### Number of uniserial *p*-adic space groups:

p = 2: Newman, O'Brien p odd: construction algorithm by Eick

	r = 2	<i>r</i> = 3	<i>r</i> = 4
p = 2	2	22	
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### **Number of pro-***p***-groups:**

• 5 pro-2-groups of coclass 2.

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## NUMBER OF PRO-*p*-GROUPS

### Number of uniserial *p*-adic space groups:

p = 2: Newman, O'Brien p odd: construction algorithm by Eick

	<i>r</i> = 2	<i>r</i> = 3	<i>r</i> = 4
<i>p</i> = 2	2	22	
<i>p</i> = 3	10	1271	137299952383
<i>p</i> = 5	95	1110136753555665	
p = 7	4575		

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## Number of pro-*p*-groups:

- 5 pro-2-groups of coclass 2,
- 54 pro-2-groups of coclass 3.

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## NUMBER OF PRO-*p*-GROUPS

## Number of uniserial *p*-adic space groups:

p = 2: Newman, O'Brien p odd: construction algorithm by Eick

	<i>r</i> = 2	<i>r</i> = 3	<i>r</i> = 4
<i>p</i> = 2	2	22	
p = 3	10	1271	137299952383
<i>p</i> = 5	95	1110136753555665	
p = 7	4575		

## Number of pro-*p*-groups:

- 5 pro-2-groups of coclass 2,
- 54 pro-2-groups of coclass 3,
- 16 pro-3-groups of coclass 2.

## COMPUTING IMMEDIATE DESCANDANTS

HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

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Input: *p*-group *G* of coclass *r* 



INVESTIGATING *p*-GROUPS BY COCLASS WITH GAP

#### ΗΕΙΚΟ DIETRICH\*. BETTINA EICK. DÖRTE FEICHTEN-SCHLAGER\*

## **COMPUTING IMMEDIATE DESCANDANTS**

**Input:** *p*-group *G* of coclass *r* Output: Immediate descendants of G

COMPUTING COCLASS TREES WITH GAP



## COMPUTING IMMEDIATE DESCANDANTS

HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup> Input: Output:

## *p*-group *G* of coclass *r* **Immediate descendants of** *G*

(i.e. all central extensions of  $C_{\rho}$  by G of coclass r)

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## COMPUTING IMMEDIATE DESCANDANTS

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**Input:** *p*-group *G* of coclass *r* 

Output: Immediate descendants of G

(i.e. all central extensions of  $C_p$  by G of coclass r)

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## Sketch of algorithm:

Let U ⊆ Z<sup>2</sup>(G, C<sub>p</sub>) correspond to the central extensions of coclass r.

## COMPUTING IMMEDIATE DESCANDANTS

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**Input:** *p*-group *G* of coclass *r* 

**Output:** Immediate descendants of *G* 

(i.e. all central extensions of  $C_{\rho}$  by G of coclass r)

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## Sketch of algorithm:

Let U ⊆ Z<sup>2</sup>(G, C<sub>p</sub>) correspond to the central extensions of coclass r.

• Consider  $U \subseteq \mathbb{F}'_p$  for some *I*.

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**Input:** *p*-group *G* of coclass *r* 

- **Output:** Immediate descendants of *G* 
  - (i.e. all central extensions of  $C_{\rho}$  by G of coclass r)

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## Sketch of algorithm:

- Let U ⊆ Z<sup>2</sup>(G, C<sub>p</sub>) correspond to the central extensions of coclass r.
- Consider U ⊆ 𝔽<sup>I</sup><sub>p</sub> for some I and let U ⊆ U be the subset of normed vectors.

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## Sketch of algorithm:

- Let U ⊆ Z<sup>2</sup>(G, C<sub>p</sub>) correspond to the central extensions of coclass r.
- Consider U ⊆ 𝔽<sup>I</sup><sub>p</sub> for some I and let U ⊆ U be the subset of normed vectors.
- The action of Comp(G, C<sub>p</sub>) = Aut(G) × Aut(C<sub>p</sub>) on U translates to an action of Aut(G) on U.

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## Sketch of algorithm:

- Let U ⊆ Z<sup>2</sup>(G, C<sub>p</sub>) correspond to the central extensions of coclass r.
- Consider U ⊆ 𝔽<sup>I</sup><sub>p</sub> for some I and let U ⊆ U be the subset of normed vectors.
- The action of Comp(G, C<sub>p</sub>) = Aut(G) × Aut(C<sub>p</sub>) on U translates to an action of Aut(G) on U.

### THEOREM

There is a 1–1 corresp. between the Aut(G)-orbits of  $\overline{U}$  and the isomorphism types of immediate descendants of G.

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## CONJECTURE FOR COHOMOLOGY

### THEOREM (CARSLON)

Let k be a field with char(k) = 2 and  $r \in \mathbb{N}$ . Then there exist only finitely many isomorphism types of cohomology rings  $H^*(G, k)$  where G is a 2-group of coclass r.

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### CONJECTURE

Let k be a field with char(k) = 2 and  $\mathcal{F}_G$  a coclass family of 2-groups.

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## CONJECTURE FOR COHOMOLOGY

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Let k be a field with char(k) = 2 and  $r \in \mathbb{N}$ . Then there exist only finitely many isomorphism types of cohomology rings  $H^*(G, k)$  where G is a 2-group of coclass r.

### CONJECTURE

Let k be a field with char(k) = 2 and  $\mathcal{F}_G$  a coclass family of 2-groups. Then  $H^*(G, k) \cong H^*(H, k)$  for all  $H \in \mathcal{F}_G$ .

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### THEOREM

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If  $i, j \ge 4$ , then

•  $H^*(D_{2^i}, \mathbb{F}_2) \cong H^*(D_{2^j}, \mathbb{F}_2).$ 

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 $\mathcal{G}(2,1)$  and cohomology

# $\mathcal{G}(2,1)$ and cohomology

HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

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If  $i, j \geq 4$ , then

**THEOREM** 

- $i, j \geq 4, inch$
- $H^*(D_{2^i}, \mathbb{F}_2) \cong H^*(D_{2^j}, \mathbb{F}_2);$
- $H^*(Q_{2^i}, \mathbb{F}_2) \cong H^*(Q_{2^j}, \mathbb{F}_2).$

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# $\mathcal{G}(2,1)$ and cohomology

HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

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- $H^*(D_{2^j}, \mathbb{F}_2) \cong H^*(D_{2^j}, \mathbb{F}_2);$
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- $H^*(SD_{2^i}, \mathbb{F}_2) \cong H^*(SD_{2^j}, \mathbb{F}_2).$

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## $\mathcal{G}(2,1)$ and cohomology

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### Furthermore,

•  $H^*(D_{2^i},\mathbb{Z})$  has a presentation depending only on *i*.

## $\mathcal{G}(2,1)$ and cohomology

HEIKO DIETRICH\*, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER\*

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### Furthermore,

- $H^*(D_{2^i},\mathbb{Z})$  has a presentation depending only on *i*;
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## $\mathcal{G}(2,1)$ and cohomology

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Conjectured mod2-cohomology for  $\mathcal{B}_{2i}$ ,  $\mathcal{B}_{2i+1} \subseteq \mathcal{T}_1(2,2)$  ( $i \in \mathbb{N}_0$ ).

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mod2-cohomology for  $\mathcal{B}_i \subseteq \mathcal{T}_2(2,2)$  ( $0 \le i \le 4$ ).



Conjectured mod2-cohomology for  $\mathcal{B}_i \subseteq \mathcal{T}_3(2,2)$   $(i \in \mathbb{N}_0)$ .





INVESTIGATING <i>p</i> -groups by coclass with	WIDTHS AND DEPTHS
GAP HEIKO	
DIETRICH <sup>*</sup> , BETTINA EICK, DÖRTE FEICHTEN- SCHLAGER <sup>*</sup>	
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## WIDTHS AND DEPTHS

HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

## We have seen that

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 $\mathcal{G}(2, r), r \ge 1$ , and  $\mathcal{G}(3, 1)$ : bounded width and depth.

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#### COCLASS WITH GAP HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK,

## WIDTHS AND DEPTHS

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SCHLAGER\*

DÖRTE

FEICHTEN-

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### and we expect that

 $\mathcal{G}(5, 1)$ : bounded width, unbounded depth.

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#### ΗΕΙΚΟ DIETRICH\*. BETTINA EICK. DÖRTE FEICHTEN-SCHLAGER\*

APPLICATION: G(5, 1)

## WIDTHS AND DEPTHS

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 $\mathcal{G}(5,1)$ : bounded width, unbounded depth.

 $\mathcal{G}(p, 1), p \geq 7$ : unbounded width and depth.

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HEIKO

## <sup>G</sup> WIDTHS AND DEPTHS

#### DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

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HEIKO DIETRICH<sup>\*</sup>.

## WIDTHS AND DEPTHS

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HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE

## UDTHS AND DEPTHS

### We have seen that

SCHLAGER\*

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### and we expect that

 $\mathcal{G}(5, 1)$ : bounded width, unbounded depth.

 $\mathcal{G}(p, 1), p \geq 7$ : unbounded width and depth.

 $\mathcal{G}(p, r), p \ge 3, r \ge 2$ : complex structure.

 $\rightsquigarrow$  Consider  $\mathcal{G}(5,1)$  in more detail.

## **GROUPS OF COCLASS 1 – NOTATION**

HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

It is known:

# • Pro-*p*-group of coclass 1 is $S = C_p \ltimes T$ with $T = \mathbb{Z}_p^{p-1}$ .

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GROUPS OF COCLASS 1 - NOTATION

HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

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HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

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- Mainline groups of  $\mathcal{T}$ :  $G_0, G_1, \ldots$  with  $G_i = S/\gamma_{i+2}(S)$ .

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HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

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- Let  $\mathcal{B}_i(k)$  be the *shaved subtree* of  $\mathcal{B}_i$  of depth k

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- Let B<sub>i</sub>(k) be the shaved subtree of B<sub>i</sub> of depth k (generated by all capable groups of depth at most k in B<sub>i</sub> and all their terminal immediate descendants).
- Let the *collar*  $\mathcal{B}_i(I, k)$  be defined as  $\mathcal{B}_i(k) \setminus \mathcal{B}_i(I-1)$ .

#### HEIKO DIETRICH\*, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER\*

### CONJECTURE

THE BRANCH  $\mathcal{B}_i$  OF  $\mathcal{G}(5, 1)$ 

Let  $i \ge 8$  and write i = 8 + 4x + y with  $0 \le y \le 3$  and  $x \ge 0$ .

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#### HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

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# The branch $\mathcal{B}_i$ of $\mathcal{G}(5,1)$

### **CONJECTURE**

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• The branch  $\mathcal{B}_i$  of  $\mathcal{G}(5, 1)$  has depth i + 2.

# The branch $\mathcal{B}_i$ of $\mathcal{G}(5,1)$

CONJECTURE

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Let  $i \ge 8$  and write i = 8 + 4x + y with  $0 \le y \le 3$  and  $x \ge 0$ .

• The branch  $\mathcal{B}_i$  of  $\mathcal{G}(5, 1)$  has depth i + 2. It consists of

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- a head  $H(i) = B_i(5+y)$ .

#### HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

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# The branch $\mathcal{B}_i$ of $\mathcal{G}(5,1)$

CONJECTURE

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- a head 
$$H(i) = B_i(5+y),$$

- *a tail*  $T(i) = B_i(i-2,i+1).$ 

#### HEIKO DIETRICH\*, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER\*

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  - a head  $H(i) = \mathcal{B}_i(5+y),$
  - $x \text{ collars } C(i,j) = B_i(6+y+4j,9+y+4j)$ with  $0 \le j \le x-1$ , and

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- *a tail*  $T(i) = B_i(i-2,i+1).$ 

# The branch $\mathcal{B}_i$ of $\mathcal{G}(5,1)$

CONJECTURE

HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

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- *a tail*  $T(i) = B_i(i-2,i+1).$ 

•  $H(i) \cong H(i+4)$  and  $T(i) \cong T(i+4)$ .

# THE BRANCH $\mathcal{B}_i$ of $\mathcal{G}(5,1)$

HEIKO DIETRICH\*, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER\*

## CONJECTURE

Let  $i \ge 8$  and write i = 8 + 4x + y with  $0 \le y \le 3$  and  $x \ge 0$ .

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- *a tail*  $T(i) = B_i(i-2,i+1).$
- *H*(*i*) ≅ *H*(*i* + 4) and *T*(*i*) ≅ *T*(*i* + 4).
   *C*(*i*, *j*) ≅ *C*(*i* + 4, *j*)

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# The branch $\mathcal{B}_i$ of $\mathcal{G}(5,1)$

HEIKO DIETRICH\*, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER\*

## CONJECTURE

Let  $i \ge 8$  and write i = 8 + 4x + y with  $0 \le y \le 3$  and  $x \ge 0$ .

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- *a tail*  $T(i) = B_i(i-2,i+1).$
- *H*(*i*) ≅ *H*(*i* + 4) and *T*(*i*) ≅ *T*(*i* + 4). *C*(*i*, *j*) ≅ *C*(*i* + 4, *j*) and *C*(*i*, *j*) ≅ *C*(*i*, *j* − 1).

COHOMOLOGY OF 2-GROUPS

**G(5, 1)** Coclass 1



Structures of  $\mathcal{B}_i, \mathcal{B}_{i+4}, \ldots$  with  $12 \leq i \leq 15$ .

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The conjectured branches  $\mathcal{B}_i$  with i = 8 + 4x + 1 and  $x \ge 0$ .



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The origins of infinite coclass families in  $\mathcal{B}_i$ ,  $12 \le i \le 15$ .



The origins of infinite coclass families in  $\mathcal{B}_i$ ,  $12 \le i \le 15$ .

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The origins of infinite coclass families in  $\mathcal{B}_i$ ,  $12 \le i \le 15$ .

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HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

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## $\mathcal{G}(5,1)$ and coclass families

### Infinite coclass families in $\mathcal{G}(5, 1)$ :

The groups in *H*(*i*), *T*(*i*), and *C*(*i*,0) with 12 ≤ *i* ≤ 15 would define disjoint infinite coclass families.

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 $\mathcal{G}(5,1)$  and coclass families

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### Infinite coclass families in $\mathcal{G}(5, 1)$ :

- The groups in *H*(*i*), *T*(*i*), and *C*(*i*,0) with 12 ≤ *i* ≤ 15 would define disjoint infinite coclass families.
- Their union would contain all groups in  $\mathcal{G}(5, 1)$  which are descendants of  $G_{12}$ .

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### Infinite coclass families in $\mathcal{G}(5, 1)$ :

- The groups in *H*(*i*), *T*(*i*), and *C*(*i*,0) with 12 ≤ *i* ≤ 15 would define disjoint infinite coclass families.
- Their union would contain all groups in  $\mathcal{G}(5, 1)$  which are descendants of  $G_{12}$ .

### Conjectured number of infinite coclass families:

# families	<i>y</i> = 0	<i>y</i> = 1	<i>y</i> = 2	<i>y</i> = 3	Σ
in the heads	366	578	741	953	2638
in the collars	748	756	748	756	3008
in the tails	730	735	730	737	2932
Σ	1844	2069	2219	2446	8578

# INVESTIGATING $\mathcal{G}(5,1)$ and Schur multiplicators *p*-GROUPS BY COCLASS WITH GAP ΗΕΙΚΟ DIETRICH\*. BETTINA EICK. DÖRTE **CONJECTURE** FEICHTEN-SCHLAGER\* For $n \in \mathbb{N}$ write $n = 4s_n + r_n$ with $1 \le r_n \le 4$ . APPLICATION: G(5, 1)

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# $\mathcal{G}(5,1)$ and Schur multiplicators

### **CONJECTURE**

For *n* ∈ N write *n* = 4*s*<sub>*n*</sub> + *r*<sub>*n*</sub> with 1 ≤ *r*<sub>*n*</sub> ≤ 4. • If *G* ∈ B<sub>*i*</sub> with *i* ≥ 8 is capable and |*G*| = *p*<sup>*n*</sup>, then I(M(G)) =  $\begin{cases} (5, 5^{s_n}, 5^{s_n}) & \text{if } r_n = 1, 2, \\ (5, 5^{s_n}, 5^{s_n+1}) & \text{if } r_n = 3, 4. \end{cases}$ 

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# $\mathcal{G}(5,1)$ and Schur multiplicators

### CONJECTURE

For *n* ∈ N write *n* = 4*s*<sub>*n*</sub> + *r*<sub>*n*</sub> with 1 ≤ *r*<sub>*n*</sub> ≤ 4. • If *G* ∈ B<sub>*i*</sub> with *i* ≥ 8 is capable and |*G*| = *p*<sup>*n*</sup>, then  $I(M(G)) = \begin{cases} (5, 5^{s_n}, 5^{s_n}) & \text{if } r_n = 1, 2, \\ (5, 5^{s_n}, 5^{s_n+1}) & \text{if } r_n = 3, 4. \end{cases}$ 

• If H is a terminal immediate descendant of G, then

$$I(M(H)) = \begin{cases} (5^{s_n}, 5^{s_n}) & \text{if } r_n = 1, 2, \\ (5^{s_n}, 5^{s_n+1}) & \text{if } r_n = 3, 4. \end{cases}$$

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# $\mathcal{G}(5,1)$ and Schur multiplicators

### **Coclass families:**

• Let  $G \in \mathcal{B}_i$  define a coclass family  $\mathcal{F}$ .

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# $\mathcal{G}(5,1)$ and Schur multiplicators

HEIKO DIETRICH<sup>\*</sup>, BETTINA EICK, DÖRTE FEICHTEN-SCHLAGER<sup>\*</sup>

### **Coclass families:**

- Let  $G \in \mathcal{B}_i$  define a coclass family  $\mathcal{F}$ .
- Let  $\mathcal{K} \in \mathcal{F}$  and  $I(M(G)) = (5^a, 5^b, 5^c), a \in \{0, 1\}.$

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# $\mathcal{G}(5, 1)$ and Schur multiplicators

ΗΕΙΚΟ DIETRICH\*. BETTINA EICK. DÖRTE FEICHTEN-SCHLAGER\*

### Coclass families:

- Let  $G \in \mathcal{B}_i$  define a coclass family  $\mathcal{F}$ .
- Let  $K \in \mathcal{F}$  and  $I(M(G)) = (5^a, 5^b, 5^c), a \in \{0, 1\}.$

### Then:

APPLICATION: G(5, 1)

 $K \in \mathcal{B}_i$  for some j with j - i = 4l,  $l \in \mathbb{N}_0$ , and we would have:

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# $\mathcal{G}(5,1)$ and Schur multiplicators

### **Coclass families:**

- Let  $G \in \mathcal{B}_i$  define a coclass family  $\mathcal{F}$ .
- Let  $K \in \mathcal{F}$  and  $I(M(G)) = (5^a, 5^b, 5^c)$ ,  $a \in \{0, 1\}$ .

### Then:

 $K \in \mathcal{B}_j$  for some j with j - i = 4I,  $l \in \mathbb{N}_0$ , and we would have:

$$I(M(K)) = \begin{cases} (5^{a}, 5^{b+i}, 5^{c+i}) & \text{if } G \in H(i), \\ \end{cases}$$

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# $\mathcal{G}(5,1)$ and Schur multiplicators

### Coclass families:

- Let  $G \in \mathcal{B}_i$  define a coclass family  $\mathcal{F}$ .
- Let  $K \in \mathcal{F}$  and  $I(M(G)) = (5^a, 5^b, 5^c)$ ,  $a \in \{0, 1\}$ .

### Then:

 $K \in \mathcal{B}_j$  for some j with j - i = 4I,  $l \in \mathbb{N}_0$ , and we would have:

$$I(M(K)) = \begin{cases} (5^{a}, 5^{b+l}, 5^{c+l}) & \text{if } G \in H(i), \\ (5^{a}, 5^{b+2l}, 5^{c+2l}) & \text{if } G \in T(i), \end{cases}$$

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#### Lumpopulation

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# $\mathcal{G}(5,1)$ and Schur multiplicators

### Coclass families:

- Let  $G \in \mathcal{B}_i$  define a coclass family  $\mathcal{F}$ .
- Let  $K \in \mathcal{F}$  and  $I(M(G)) = (5^a, 5^b, 5^c)$ ,  $a \in \{0, 1\}$ .

### Then:

 $K \in \mathcal{B}_j$  for some j with j - i = 4I,  $l \in \mathbb{N}_0$ , and we would have:

$$I(M(K)) = \begin{cases} (5^{a}, 5^{b+l}, 5^{c+l}) & \text{if } G \in H(i), \\ (5^{a}, 5^{b+2l}, 5^{c+2l}) & \text{if } G \in T(i), \\ (5^{a}, 5^{b+l+k}, 5^{c+l+k}) & \text{if } G \in C(i, 0), K \in C(j, k). \end{cases}$$

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# $\mathcal{G}(5,1)$ and outer automorphism groups

### CONJECTURE

Let  $\mathcal{F}$  be an infinite coclass family. Let  $G \in \mathcal{F} \cap \mathcal{B}_i$  with  $i \geq 12$ .

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# $\mathcal{G}(5,1)$ and outer automorphism groups

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**CONJECTURE** 

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Let  $\mathcal{F}$  be an infinite coclass family. Let  $G \in \mathcal{F} \cap \mathcal{B}_i$  with  $i \ge 12$ . Then there exist u and v depending on  $\mathcal{F}$  (but not on G or i) with

 $|Out(G)|=5^{i+\nu}u.$ 

# $\mathcal{G}(5,1)$ and outer automorphism groups

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Let  $\mathcal{F}$  be an infinite coclass family. Let  $G \in \mathcal{F} \cap \mathcal{B}_i$  with  $i \ge 12$ . Then there exist u and v depending on  $\mathcal{F}$  (but not on G or i) with

 $|Out(G)| = 5^{i+v}u.$ 

### Moreover,

**CONJECTURE** 

- *if* F *arises from a head, then*  $u \in \{1, 2, 4, 16\}$  *and*  $v \in \{-1, 0, 1, 2, 3\}$ .
- if  $\mathcal{F}$  arises from a tail or collar, then  $u \in \{1, 2, 4\}$  and  $v \in \{2, 3\}$ .

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### CONSTRUCT GROUPS OF COCLASS 1

• Let p > 3 be a prime.

Recall:  $S = C_p \ltimes T$  and  $G_i = S/\gamma_{i+2}(S)$ .

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• Let  $\overline{\mathcal{B}}_i = \mathcal{B}_i(i - 2p + 6)$ . Then:

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### CONSTRUCT GROUPS OF COCLASS 1

• Let p > 3 be a prime.

Recall:  $S = C_{\rho} \ltimes T$  and  $G_i = S/\gamma_{i+2}(S)$ .

• Let 
$$\overline{\mathcal{B}}_i = \mathcal{B}_i(i-2p+6)$$
. Then:

•  $H \in \overline{B}_i$  is an extension of  $A = T/\gamma_j(S)$  for some *j* by the root of  $\overline{B}_i$ , the mainline group  $G_i$ .

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• It is  $H^2(G_i, A) \cong twig \times ext \times hom$  with

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### CONSTRUCT GROUPS OF COCLASS 1

Let *p* > 3 be a prime.
 Recall: *S* = *C*<sub>*p*</sub> κ *T* and *G*<sub>*i*</sub> = *S*/γ<sub>*i*+2</sub>(*S*).

• Let 
$$\overline{\mathcal{B}}_i = \mathcal{B}_i(i-2p+6)$$
. Then:

- $H \in \overline{B}_i$  is an extension of  $A = T/\gamma_j(S)$  for some *j* by the root of  $\overline{B}_i$ , the mainline group  $G_i$ .
- It is  $H^2(G_i, A) \cong twig \times ext \times hom$  with

twig 
$$\cong$$
  $C_p^3$ ,

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- It is  $H^2(G_i, A) \cong twig \times ext \times hom$  with

$$\begin{array}{rcl} twig &\cong& C_{\rho}^{3},\\ ext &\cong& A, \end{array}$$

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- It is  $H^2(G_i, A) \cong twig \times ext \times hom$  with

twig 
$$\cong C_{\rho}^{3}$$
,  
ext  $\cong A$ , and  
hom  $\cong A^{\frac{p-3}{2}}$ .

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### CONSTRUCT GROUPS OF COCLASS 1

### Which elements of $H^2(G_i, A)$ induce extensions in $\overline{\mathcal{B}}_i$ ?

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### CONSTRUCT GROUPS OF COCLASS 1

### Which elements of $H^2(G_i, A)$ induce extensions in $\overline{\mathcal{B}}_i$ ?

• For  $\gamma \in H^2(G_i, A)$  let  $E(\gamma)$  be the corresp. extension.

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• Fix  $\varepsilon \in ext$  with  $E(\varepsilon)$  of coclass 1 ( $\rightsquigarrow$  mainline).

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• Fix  $\varepsilon \in ext$  with  $E(\varepsilon)$  of coclass 1 ( $\rightsquigarrow$  mainline).

•  $H \in \overline{\mathcal{B}}_i \iff \varepsilon + \tau + \kappa$  where  $\tau \in twig$  and  $\kappa \in hom$ with  $\kappa \notin H^2(G_i, \gamma_2(S)/\gamma_j(S))$ .

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• Fix  $\varepsilon \in ext$  with  $E(\varepsilon)$  of coclass 1 ( $\rightsquigarrow$  mainline).

•  $H \in \overline{\mathcal{B}}_i \quad \leftrightarrow \quad \varepsilon + \tau + \kappa \text{ where } \tau \in twig \text{ and } \kappa \in hom$ with  $\kappa \notin H^2(G_i, \gamma_2(S)/\gamma_j(S)).$ 

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•  $E(\varepsilon + \tau_1 + \kappa_1) \cong E(\varepsilon + \tau_2 + \kappa_2)$  iff

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- For  $\gamma \in H^2(G_i, A)$  let  $E(\gamma)$  be the corresp. extension.
- Fix  $\varepsilon \in ext$  with  $E(\varepsilon)$  of coclass 1 ( $\rightsquigarrow$  mainline).
- $H \in \overline{\mathcal{B}}_i \quad \leftrightarrow \quad \varepsilon + \tau + \kappa \text{ where } \tau \in twig \text{ and } \kappa \in hom$ with  $\kappa \notin H^2(G_i, \gamma_2(S)/\gamma_j(S)).$
- $E(\varepsilon + \tau_1 + \kappa_1) \cong E(\varepsilon + \tau_2 + \kappa_2)$  iff  $(\tau_1 + \kappa_1)^c = \tau_2 + \kappa_2$ for some  $c \in \Sigma$  with  $\Sigma = \{(\alpha|_{S_i}, \alpha|_A) \mid \alpha \in Aut(S)\}.$

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INVESTIGATING

p-groups by

COCLASS WITH

GAP
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### CONSTRUCT GROUPS OF COCLASS 1

### Which elements of $H^2(G_i, A)$ induce extensions in $\overline{\mathcal{B}}_i$ ?

• For  $\gamma \in H^2(G_i, A)$  let  $E(\gamma)$  be the corresp. extension.

• Fix  $\varepsilon \in ext$  with  $E(\varepsilon)$  of coclass 1 ( $\rightsquigarrow$  mainline).

• 
$$H \in \overline{\mathcal{B}}_i \quad \leftrightarrow \quad \varepsilon + \tau + \kappa \quad \text{where } \tau \in twig \text{ and } \kappa \in hom$$
  
with  $\kappa \notin H^2(G_i, \gamma_2(S)/\gamma_j(S)).$ 

•  $E(\varepsilon + \tau_1 + \kappa_1) \cong E(\varepsilon + \tau_2 + \kappa_2)$  iff  $(\tau_1 + \kappa_1)^c = \tau_2 + \kappa_2$ for some  $c \in \Sigma$  with  $\Sigma = \{(\alpha|_{S_i}, \alpha|_A) \mid \alpha \in \operatorname{Aut}(S)\}.$ 

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### $\rightsquigarrow$ Compute $\Sigma$ -orbits.

# $_{\text{Y}}^{\text{NG}}$ Construct groups of coclass 1

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### So far, this theory leads to ...

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CONSTRUCT GROUPS OF COCLASS 1

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### So far, this theory leads to ...

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• The isomorphism  $H^2(G_i, A) \to H^2(G_{i+p-1}, A)$  is a  $\Sigma$ -module isomorphism.

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CONSTRUCT GROUPS OF COCLASS 1

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• The graph  $\overline{\mathcal{B}}_i$  can be embedded into  $\overline{\mathcal{B}}_{i+p-1}$ .

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- The isomorphism  $H^2(G_i, A) \to H^2(G_{i+p-1}, A)$  is a  $\Sigma$ -module isomorphism.
- The graph  $\overline{\mathcal{B}}_i$  can be embedded into  $\overline{\mathcal{B}}_{i+p-1}$ .
- Groups which correspond under this embedding have the same parametrized presentation.
CONSTRUCT GROUPS OF COCLASS 1

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 $\rightsquigarrow$  First step to prove the main conjecture for groups of coclass 1.

INVESTIGATING *p*-groups by Coclass with GAP

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## Thank you for your kind attention.

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